Write-up 10: Using Parametric Equations to Graph Line Segments

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This write-up is in response to the following prompt:

Write parametric equations of a line segment through (7,5) with slope of 3. Graph the line segment using your equations. As a line segment, it will have end points. Explore how you would choose endpoints of such that the two distances from (7,5) are 2 units and 3 units.

Having rarely used parametric equations in my mathematics classes, this was an interesting write-up for me. I started writing my equations using the point my line must go through as an origin of sorts and a consideration of slope as "rise over run":
After this successful first attempt, I moved on to the second part of the prompt: choosing

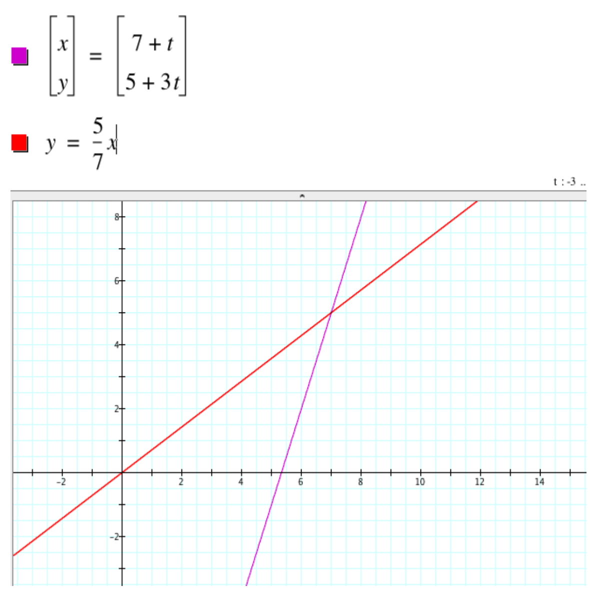
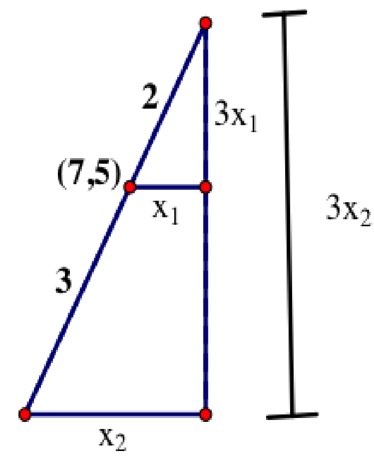


Figure 1: The line $y = \frac{5}{7}x$ was used to check that the parametric equations were correct.

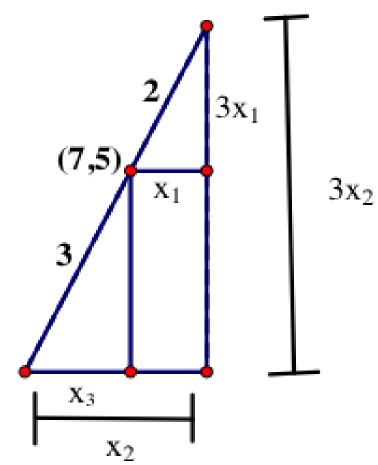
endpoints that are 2 and 3 units away from (7,5).
I first created the following diagram: Then, using the Pythagorean Theorem, I determined



$$x_1^2 + (3x_1)^2 = 4 \Rightarrow x_1 = \frac{2\sqrt{10}}{10}$$

$$x_1^2 + (3x_1)^2 = 25 \Rightarrow x_2 = \frac{5\sqrt{10}}{10}$$

I then edited my graph to include from which I could gather that



$$x_1 = \frac{2\sqrt{10}}{10},\ x_3 = \frac{3\sqrt{10}}{10}$$

Therefore, the most simple change to the parametrized line was to simply change adding t from any range to adding $\frac{\sqrt{10}}{10}t$ on the range $[-3, 2]$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 + \frac{\sqrt{10}}{10} t \\ 5 + 3\frac{\sqrt{10}}{10} t \end{bmatrix}$$

t : -3 ... 2

Could we parametrize this segment differently, though? Perhaps we could work backwards to determine what the endpoints that are 2 and 3 units away from (7,5) are *before* parametrizing the general line through (7,5). With all of the legwork done above, we can identify the endpoints as

$$(7 + \frac{2\sqrt{10}}{10}, 5 + \frac{6\sqrt{10}}{10}) \text{ and } (7 - \frac{3\sqrt{10}}{10}, 5 - \frac{9\sqrt{10}}{10})$$

Thus, if we use the lower left point as our origin, we can calculate the differences in the x and y directions as

$$x_2 = \frac{5\sqrt{10}}{10},\ 3x_2 = \frac{15\sqrt{10}}{10}$$

and parametrize as follows for $t \in [0, \frac{5\sqrt{10}}{10}]$, with $\frac{5\sqrt{10}}{10} \approx 1.58113883$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \left(7 - 3\frac{\sqrt{10}}{10}\right) + t \\ \left(5 - 9\frac{\sqrt{10}}{10}\right) + 3t \end{bmatrix}$$

t : 0 ... 1.518113883